

Webinar 2: Bridges

2.3: Structural analysis for bridges accounting for spatial variability of ground motion

Anastasios Sextos

Member of PT6

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Definition

EN 1998-2: 3.1.2

spatial variability (of seismic action)

situation in which the ground motion at different supports of the bridge differs and, hence, the seismic action cannot be based on the characterisation of the motion at a single point



Can we realistically consider that earthquake excitation is identical across the the bridge supports?

1 Wave passage/ Travelling effect

different earthquake motions excite
at each support
(in terms of amplitude,
phase and
frequency content)

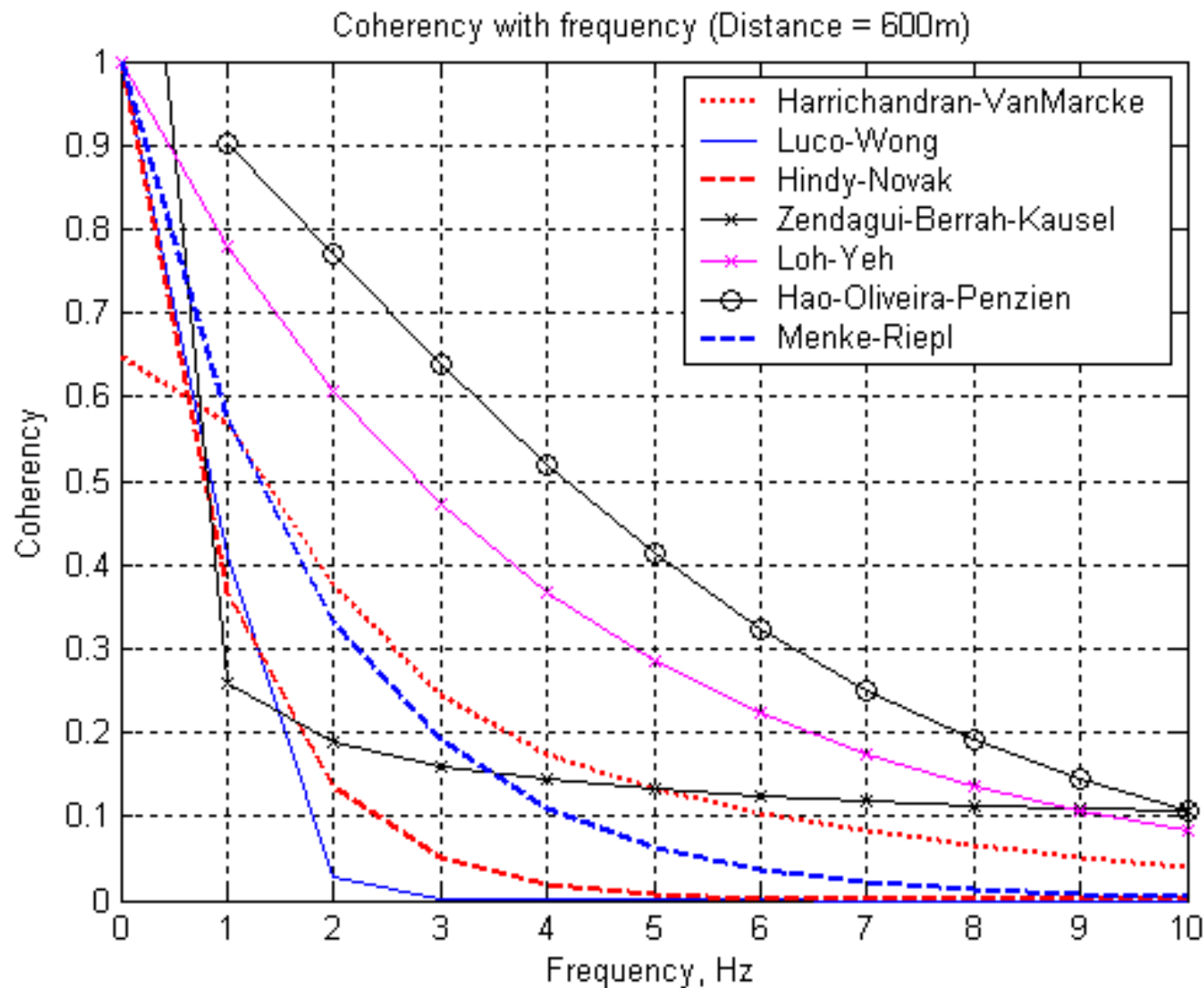
$$\theta_{jk}(\omega) = \tan^{-1} (\text{Im}[y_{jk}(\omega)] / \text{Re}[y_{jk}(\omega)])$$

2

Incoherency due to wave-scattering

$$|\bar{\gamma}_{jk}^M(\omega)| = \frac{|\bar{S}_{jk}^M(\omega)|}{\sqrt{\bar{S}_{jj}^M(\omega)\bar{S}_{kk}^M(\omega)}}$$





Free field recordings at strong motion accelerometric arrays

Different functions prescribe different rates of coherency loss with distance and frequency

3

Incoherency due to local site conditions

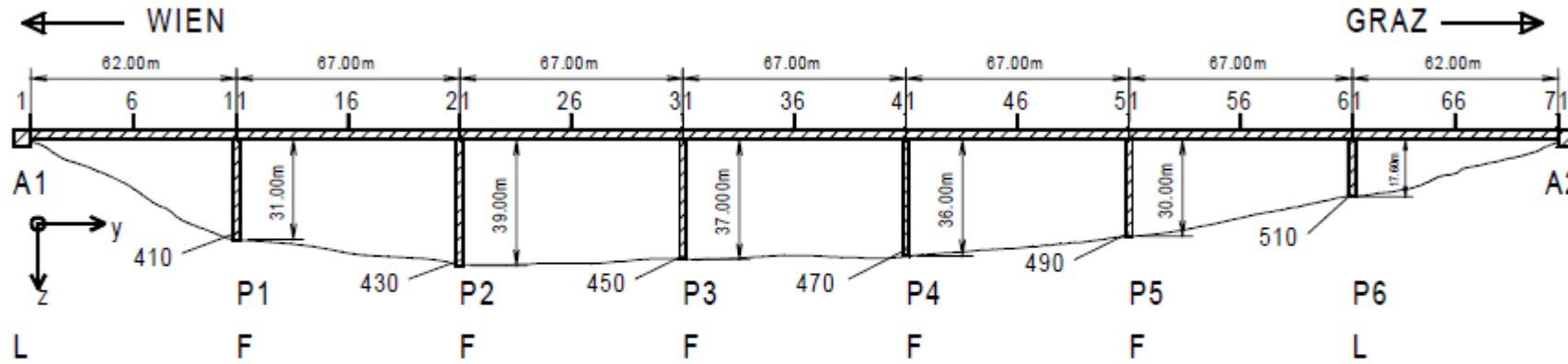


4 Non-uniform Liquefaction along bridge length

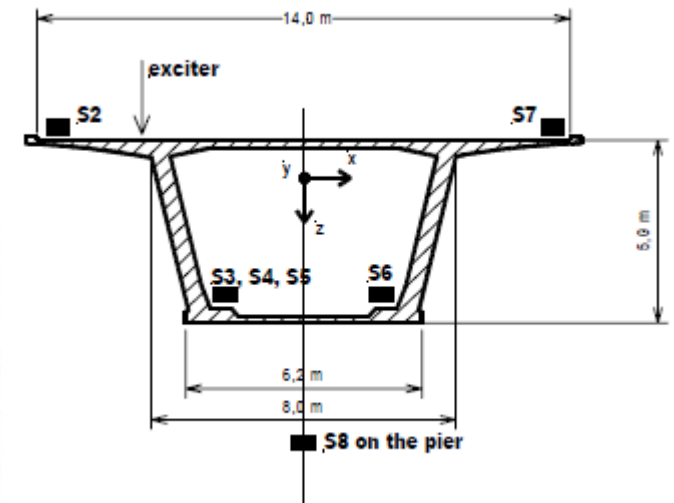


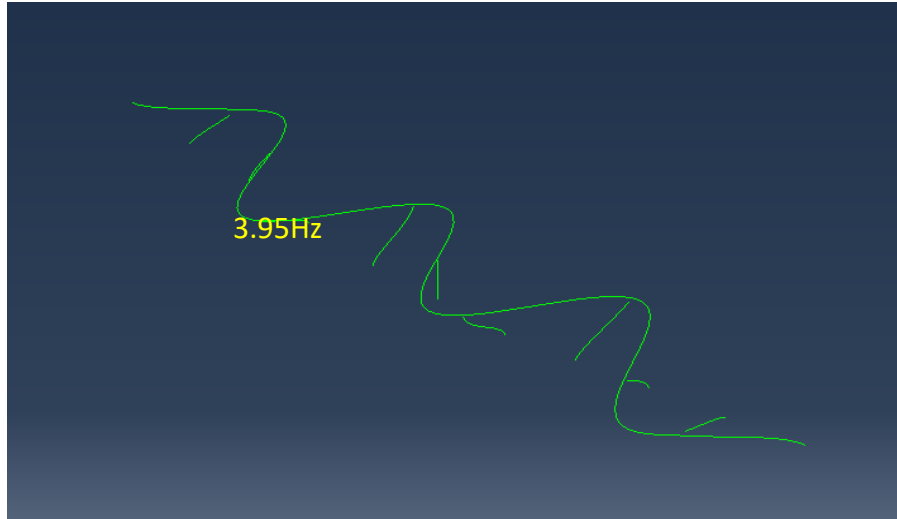
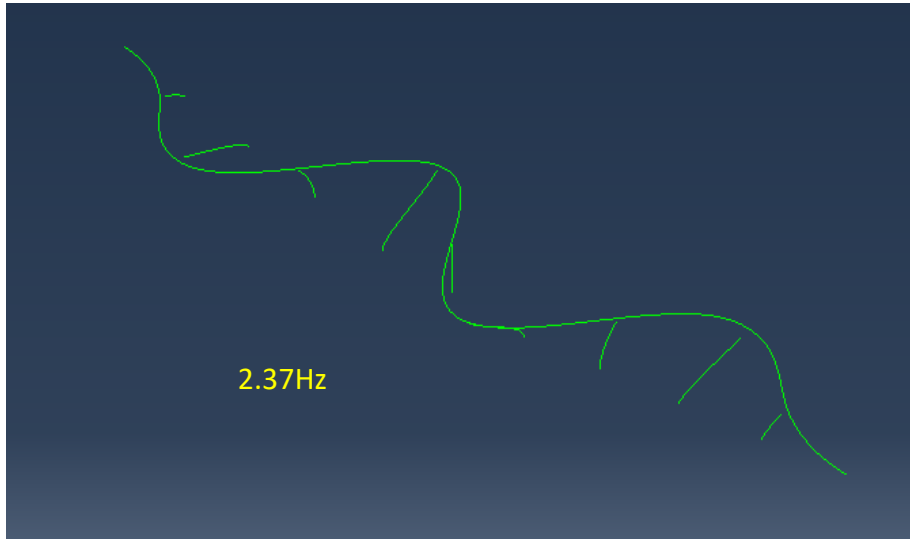
Metsovo bridge

Talübergang Warth bridge (VAB Project, PI: R. Flesch)

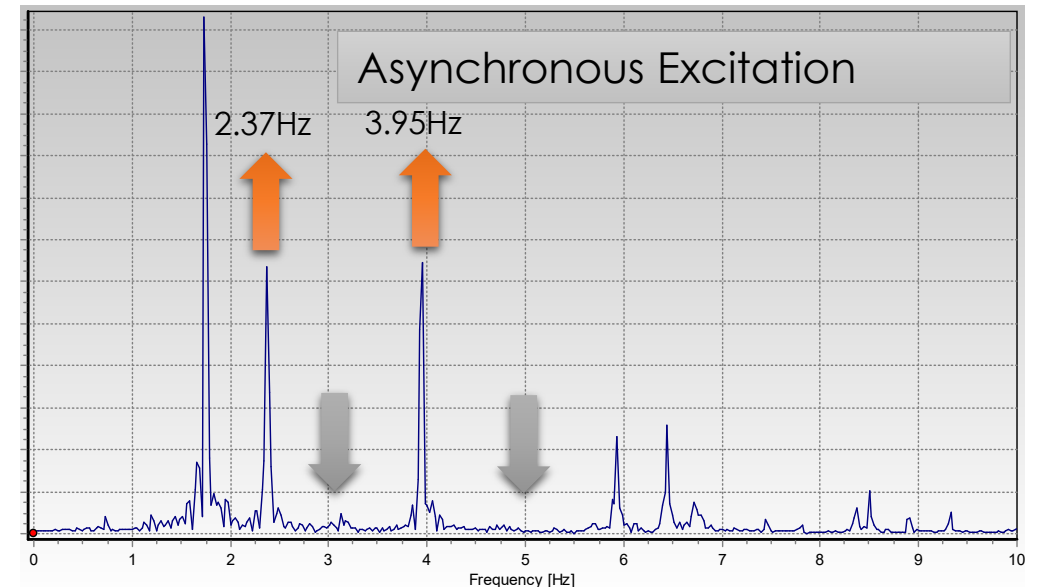
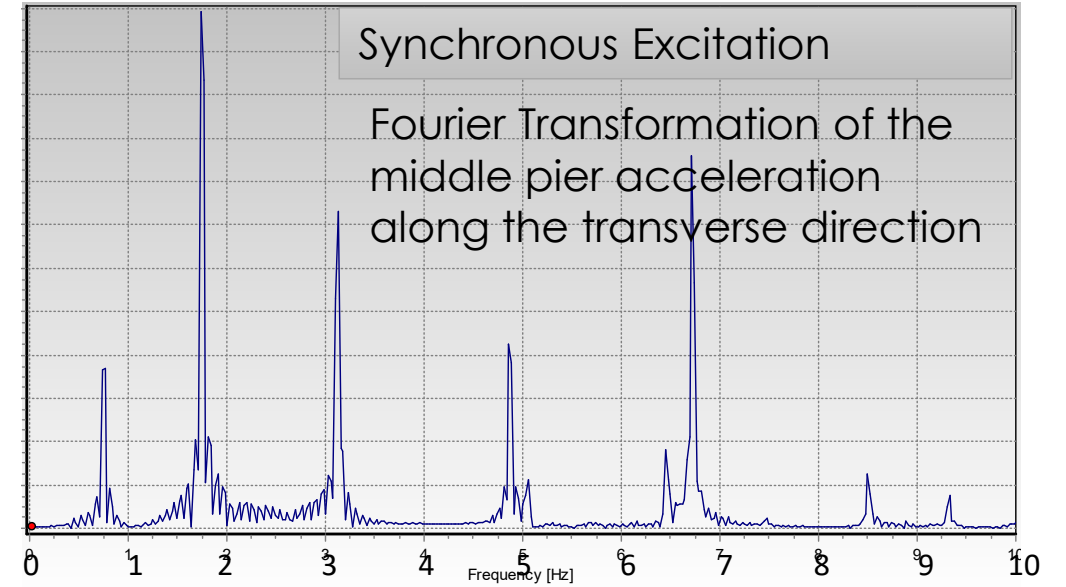


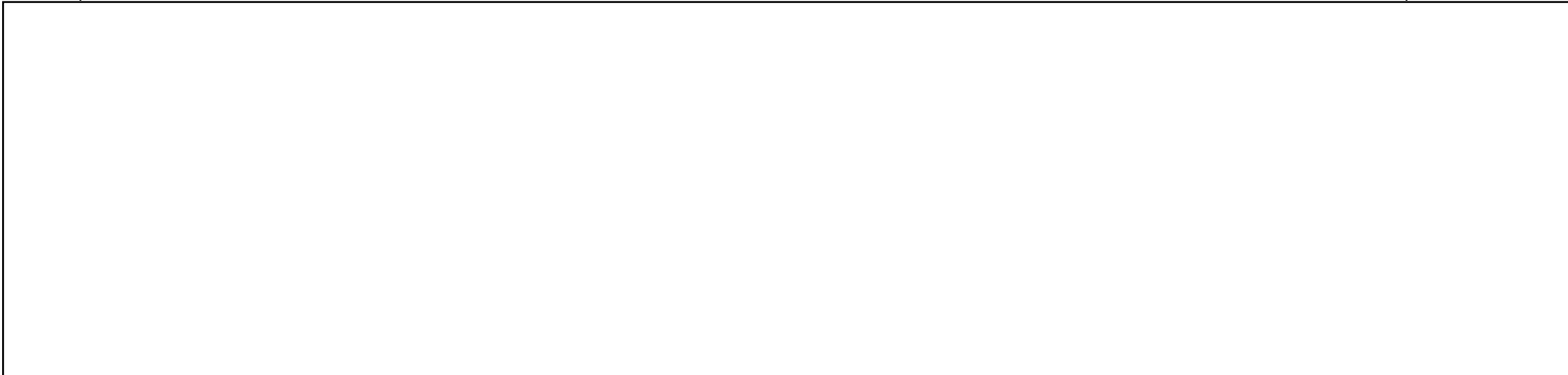
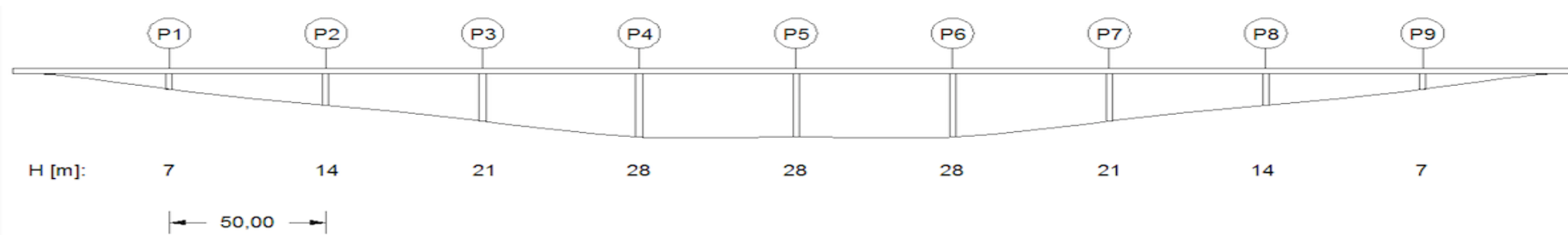
bearings:
 L.....movable, longitudinal
 F.....not movable



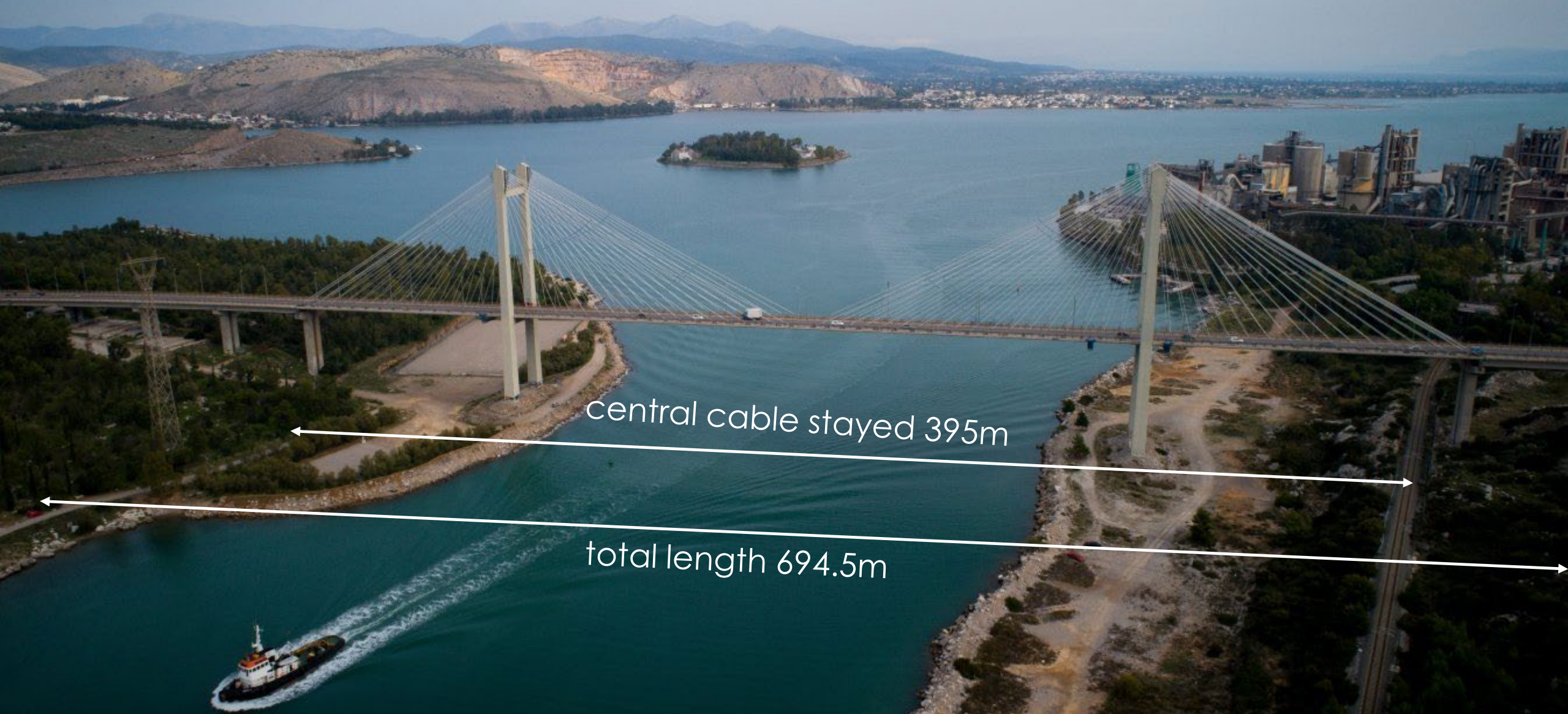


Propagation of excitation forces lock the bridge in different mode shapes



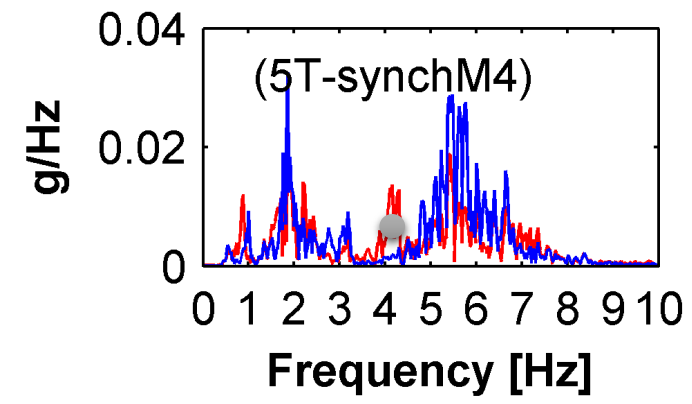
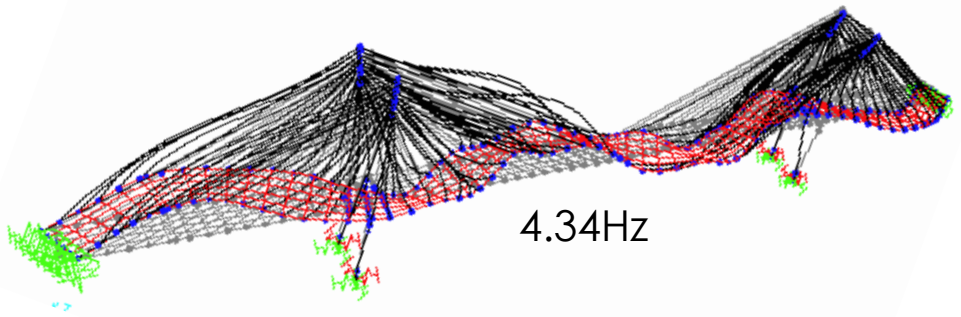
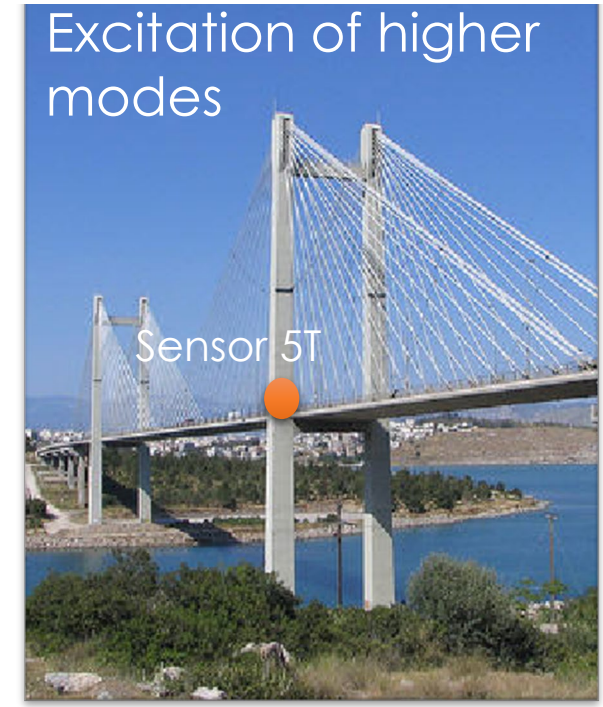
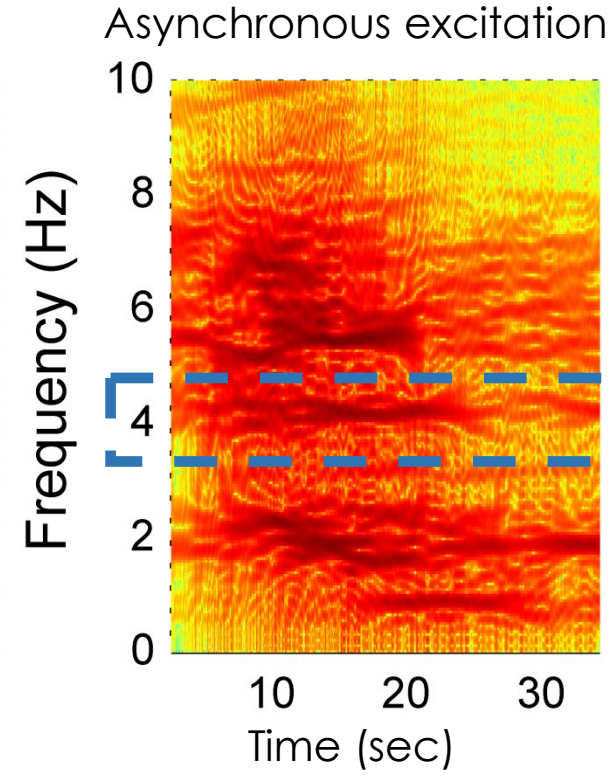
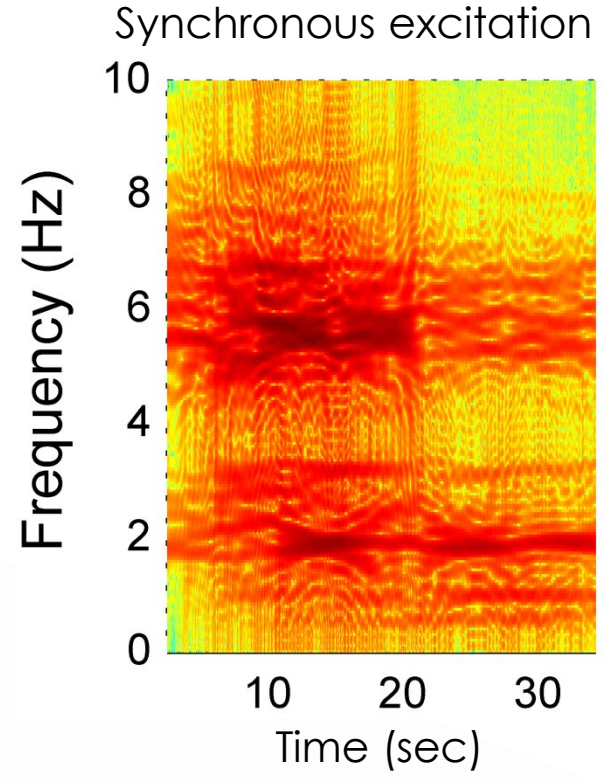


Evripos bridge
Chalkida, Greece



central cable stayed 395m

total length 694.5m



Previous Eurocode 8 provisions for spatially variable seismic action

- The possibility to account for spatial variability effects was mentioned in EN1998-1:2005, without any further development
- a quantification of the phenomenon was introduced in EN1998-2:2005.
- A simplified approach was also presented in EN1998-2:2005

Previous seismic code framework: Eurocode 8 – Part 2 (2005)

For a **soil-dependent limit length** of the continuous deck

Ground Type	A	B	C	D	E
L_g (m)	600	500	400	300	500
$L_{lim} = L_g/1.5$ (m)	400	333	266	200	333



When to
account
for multiple
support
excitation?

SVGM shall always be accounted for in cases of soils changing two or more classes along the bridge length

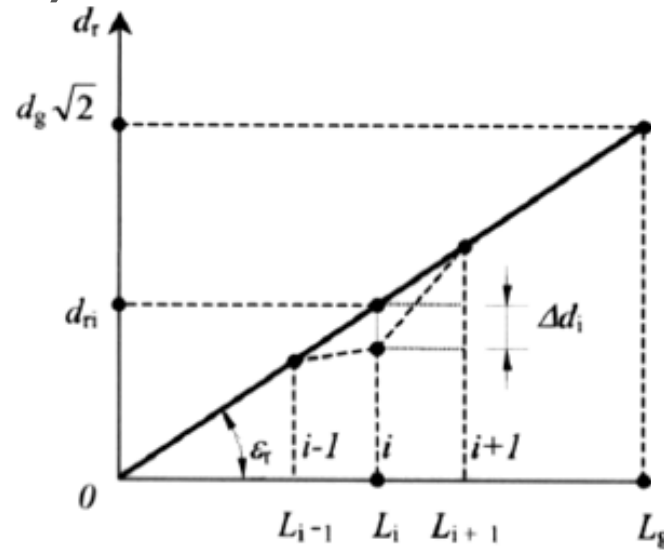
Previous seismic code framework:
Eurocode 8 – Part 2 (2005)

$$d_{ri} = e_r L_i \leq d_g \sqrt{2}$$

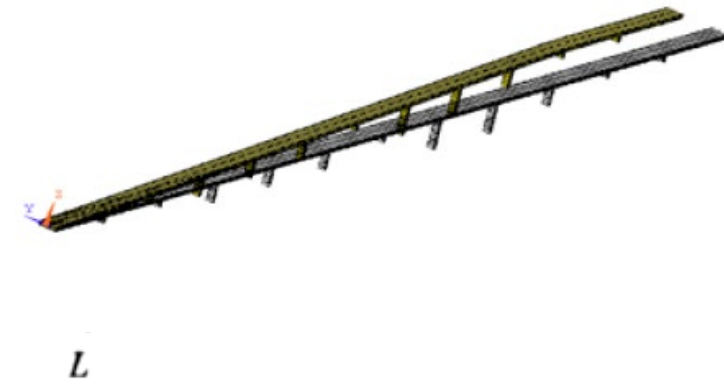
$$e_r = d_g \sqrt{2} / L_g$$

$$d_g = 0.025 \cdot a_g \cdot S \cdot T_C \cdot T_D$$

Displacement set A



$$\Delta d_i = \pm \beta_r \epsilon_r L_{av,i}$$



Displacement set B



$$\Delta d_i = \pm \beta_r e_r L_{av,i}$$

$$\rho_i = \frac{\sqrt{M_{i,SetA}^2 + M_{i,SetB}^2 + M_{i,inertial}^2}}{M_{i,inertial}}$$

Previous seismic code framework: Eurocode 8 – Part 2 (2005)

ANNEX D (INFORMATIVE) SPATIAL VARIABILITY OF EARTHQUAKE GROUND MOTION: MODEL AND METHODS OF ANALYSIS

- Time history analysis
(multiple support inputs)

D.2 Generation of samples →

$$a_i(t) = 2 \sum_{i=1}^i \sum_{k=1}^N |L_{ij}(\omega_k)| \sqrt{\Delta\omega} \cos[\omega_k t - \theta_{ij}(\omega_k) + \phi_{jk}]$$

- Multiple response spectrum solution →

D3.4.3

$$\mu_{z_{\max}} = \sqrt{\sum_{k=1}^m \sum_{l=1}^m a_k a_l \rho_{u_k u_l} u_{k,\max} u_{l,\max} + \sum_{k=1}^m \sum_{l=1}^m \sum_{i=1}^n \sum_{j=1}^n b_{ki} b_{lj} \rho_{s_{ki} s_{lj}} D_k(\omega_i, \xi_i) D_l(\omega_j, \xi_j)} \quad (D.19)$$

where $u_{k,\max}$ and $u_{l,\max}$ are the peak ground displacements at supports k and l consistent with the respective local elastic response spectrum as given in EN 1998-1:2004, **3.2.2.4**; $D_k(\omega_i, \xi_i)$ and $D_l(\omega_j, \xi_j)$ are the elastic displacement response spectra values at supports k and l for frequencies and damping ratios of the considered modes, consistent with the respective local elastic response spectrum as given in EN 1998-1:2004, **3.2.2.2**.

New Eurocode 8 provisions for spatially variable ground motions

Seismic Action Provisions EC8-1-1

EUROPEAN STANDARD
NORME EUROPÉENNE
EUROPÄISCHE NORM

DRAFT
prEN 1998-1-1

September 2022

ICS 91.010.30; 91.120.25 Will supersede EN 1998-1:2004

English Version

Eurocode 8 - Design of structures for earthquake resistance - Part 1-1: General rules and seismic action

Eurocode 8 - Calcul des structures pour leur résistance aux séismes - Partie 1-1 : Règles générales et action sismique

Eurocode 8 - Auslegung von Bauwerken gegen Erdbeben - Teil 1-1: Grundlagen und Erdbebeneinwirkung

This draft European Standard is submitted to CEN members for enquiry. It has been drawn up by the Technical Committee CEN/TC 250.


If this draft becomes a European Standard, CEN members are bound to comply with the CEN/CENELEC Internal Regulations which stipulate the conditions for giving this European Standard the status of a national standard without any alteration.

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5.2.3.2 Spatial model of the seismic action

Bridge-specific Provisions EC8-2

EUROPEAN STANDARD
NORME EUROPÉENNE
EUROPÄISCHE NORM

DRAFT
prEN 1998-2

March 2023

ICS 91.120.25; 93.040 Will supersede EN 1998-2:2005

English Version

Eurocode 8 - Design of structures for earthquake resistance - Part 2: Bridges

Eurocode 8 - Calcul des structures pour leur résistance aux séismes - Partie 2: Ponts

Eurocode 8 - Auslegung von Bauwerken gegen Erdbeben - Teil 2: Brücken

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4.2.2 Spatial variability of the seismic action

5.3 Methods of analysis accounting for spatial variability of ground motion

When to account for spatially variable ground motion?

EN1998-2 4.2.2 Spatial variability of the seismic action

(1) Spatial variability of earthquake ground motion should be considered according to Table 5.3, when any of the conditions in a) to c) holds:

- a) The maximum and minimum values of the average shear wave velocity $V_{s,H}$ calculated for the soil profiles under each bridge support (piers and abutments) differ by more than 200 m/s.
- b) The total length of the bridge exceeds L_{lim} , equal to the smallest characteristic length value L_g among all bridge supports as given in prEN 1998-1-1:2022, 5.2.3.2(3).
- c) The maximum span length between two successive supports exceeds 60 m (for bridges having two spans or more).

EN1998-1-1 5.2.3.2 Spatial model of the seismic action

Table 5.7 — Characteristic Lengths

Site category	A	B	C	D	E	F
L_g (m)	400	300	250	200	300	200

5.2.3.2 Spatial model of the seismic action

EN1998-2

(1) For structures for which the assumption of the same excitation at all supports cannot reasonably be made, spatial models of the seismic action should be used.

NOTE Allowance for the variation of ground motion in space as well as time can be required for specific types of structures (see EN 1998-2 and EN 1998-4).

(2) At every support of structures considered in (1), the seismic action should be represented according to 5.2.2, either in the form of response spectra or of time series, duly accounting for the possible site condition variability at different supports.

(3) If the seismic action is represented by a set of response spectra, for every couple of supports with indices k and l , a correlation coefficient, ρ_{kl} , between action effects may be calculated according to Formula (5.29).

$$\rho_{kl} = \exp\left(-\frac{2 L_{kl}}{a_{kl}(L_{g,k}+L_{g,l})}\right) \quad (5.29)$$

where

L_{kl} is the distance between supports k and l ;

$L_{g,k}$ and $L_{g,l}$ are characteristic lengths given in Table 5.7 as functions of the site category of the considered supports k and l , respectively;

a_{kl} is given by Formula (5.30).

$$\rho_{kl} = \exp\left(-\frac{2 L_{kl}}{a_{kl}(L_{g,k}+L_{g,l})}\right)$$

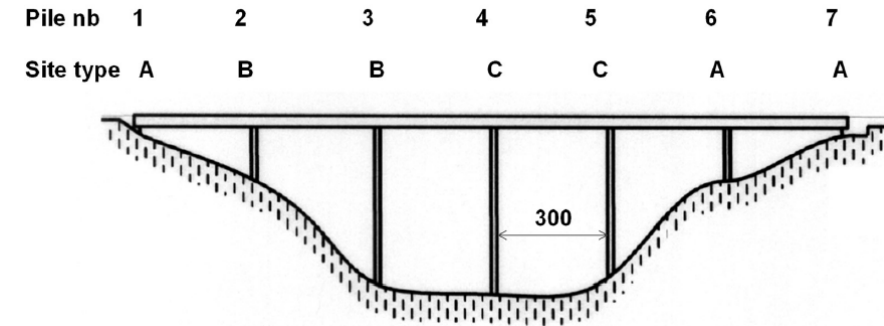
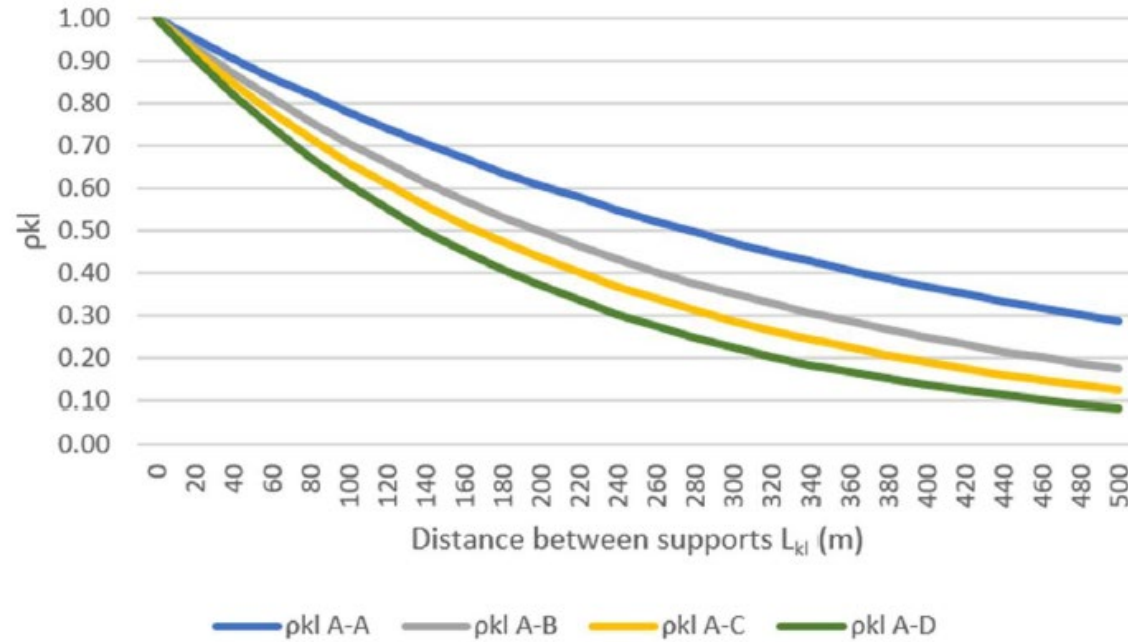
$$a_{kl} = \exp\left(-\frac{L_{g,k}-L_{g,l}}{500}\right) \rightarrow \text{accounts for a faster correlation drop for non-uniform soils.} \quad (5.30)$$

Table 5.7 — Characteristic Lengths

Site category	A	B	C	D	E	F
L_g (m)	400	300	250	200	300	200

(4) In case the seismic action is represented in the form of time series for response-history analysis, the normalized cross-correlation coefficient between two input motions applied in the same direction at supports k and l should not exceed the largest of ρ_{kl} given by Formula (5.29) and 0,2.

Characteristic bridge lengths beyond which SVGM needs to be accounted for



support	1	2	3	4	5	6	7
1	1	0,35	0,12	0,02	0,01	0,02	0,01
2	0,35	1	0,37	0,09	0,03	0,02	0,01
3	0,12	0,37	1	0,3	0,09	0,04	0,02
4	0,02	0,09	0,3	1	0,3	0,08	0,02
5	0,01	0,03	0,09	0,3	1	0,29	0,08
6	0,02	0,02	0,04	0,08	0,29	1	0,47
7	0,01	0,01	0,02	0,02	0,08	0,47	1

Fig. 1 Example of ρ_{kl} variation with distance and soil difference

Source: EN1998-1-1 BD Document, Prof. Pierre Labbe

Table 5.7 — Characteristic Lengths

Site category	A	B	C	D	E	F
L_g (m)	400	300	250	200	300	200

Table 5.3 – Analysis type for multiple support excitation

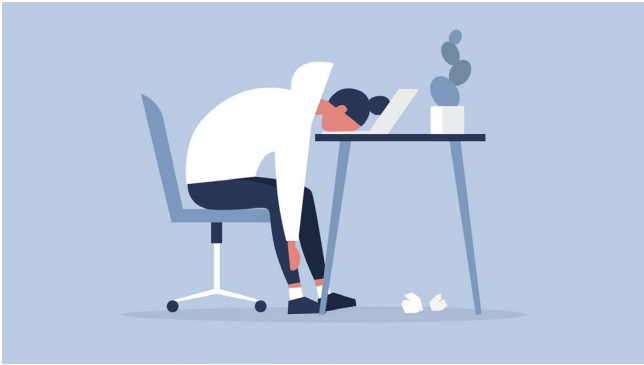
Soil conditions	Bridge and span length	
	Short bridge AND short span	Long bridge OR long span
	Short-medium length ($L \leq L_{lim}$) and maximum span between adjacent piers $L_{kl} < 60$ m	Long bridge ($L > L_{lim}$) or maximum span between two successive piers $L_{kl} > 60$ m [for bridges having two spans or more)
<p>The maximum and minimum shear wave velocity $V_{s,H}$ of the soil profiles under the supports (piers and abutments) do not vary by more than 200 m/s</p> <p>Uniform soil conditions</p>	<p>Account for spatial variability <u>is not required</u></p> 	<p><u>Simplified higher mode excitation method (5.3.1)</u></p> <p>Alternatively, the simplified higher mode excitation method of analysis can be omitted, with an application of a 20% increase in all seismic action effects* obtained from a regular uniform excitation analysis (e.g. response spectrum method)</p>

Table 5.3 – Analysis type for multiple support excitation

Soil conditions	Bridge and span length	
	Short-medium length ($L \leq L_{lim}$) and maximum span between adjacent piers $L_{kl} < 60$ m Short bridge AND short span	Long bridge ($L > L_{lim}$) or maximum span between two successive piers $L_{kl} > 60$ m (for bridges having two spans or more) Long bridge OR long span
The maximum and minimum shear wave velocity $V_{s,H}$ of the soil profiles under the supports (piers and abutments) vary by more than 200 m/s and the depth of valley along the bridge $h < 100$ m. Non-uniform soil conditions but no 2D/3D site effects	Simplified multiple-support response-history analysis, with ground motions at the supports obtained from a common input at the bedrock and separate 1D site response analyses at each support (5.3.2) Multiple 1D site response analysis (wave passage not significant, problem dominated by local soil conditions)	<u>Multiple-support response-history analysis, with ground motions that comply with the spatial variability model (5.3.3(1))</u> or <u>Multiple-support response spectrum method (5.3.3(2))</u> Proper SVGM analysis

Table 5.3 – Analysis type for multiple support excitation

Soil conditions	Bridge and span length	
	Short-medium length ($L \leq L_{lim}$) and maximum span between adjacent piers $L_{kl} < 60$ m Short bridge AND short span	Long bridge ($L > L_{lim}$) or maximum span between two successive piers $L_{kl} > 60$ m [for bridges having two spans or more] Long bridge OR long span
The maximum and minimum shear wave velocity $V_{s,H}$ of the soil profiles under the supports (piers and abutments) vary by more than 200 m/s and the depth of valley along the bridge $h < 100$ m. Non-uniform soil conditions but no 2D/3D site effects	Alternatively, 1D site response analysis can be omitted, with an application of a 20% increase in all seismic action effects* obtained from a regular uniform excitation analysis (e.g. response spectrum method) Exceptions	Alternatively, multi-support response analyses can be omitted, with an application of a 30% increase in all seismic action effects* obtained from a regular uniform excitation analysis (e.g. response spectrum method) Exceptions

Table 5.3 – Analysis type for multiple support excitation

Soil conditions	Bridge and span length	
	Short-medium length ($L \leq L_{lim}$) and maximum span between adjacent piers $L_{kl} < 60$ m Short bridge AND short span	Long bridge ($L > L_{lim}$) or maximum span between two successive piers $L_{kl} > 60$ m [for bridges having two spans or more] Long bridge OR long span
Rigorous 2D/3D Site Response Analysis		
The maximum and minimum shear wave velocity $V_{s,H}$ of the soil profiles under the supports (piers and abutments) vary by more than 200 m/s and the depth of valley along the bridge $h \geq 100$ m Non-uniform soil conditions AND 2D/3D site effects	Response-history analysis with spatially variable ground motions produced by means of 2D/3D site response analysis or Exceptions Alternatively, 2D/3D site response analysis can be omitted, with an application of a 1D site response analysis per support and an additional 30 % increase in all seismic action effects* obtained from a regular uniform excitation analysis (e.g. response spectrum method)	
* Seismic action effects include generalised stresses as well as generalised deformations. The latter include relative displacements at deck joints and supports, which should be increased to avoid unseating failure due to spatially varying ground motions.		

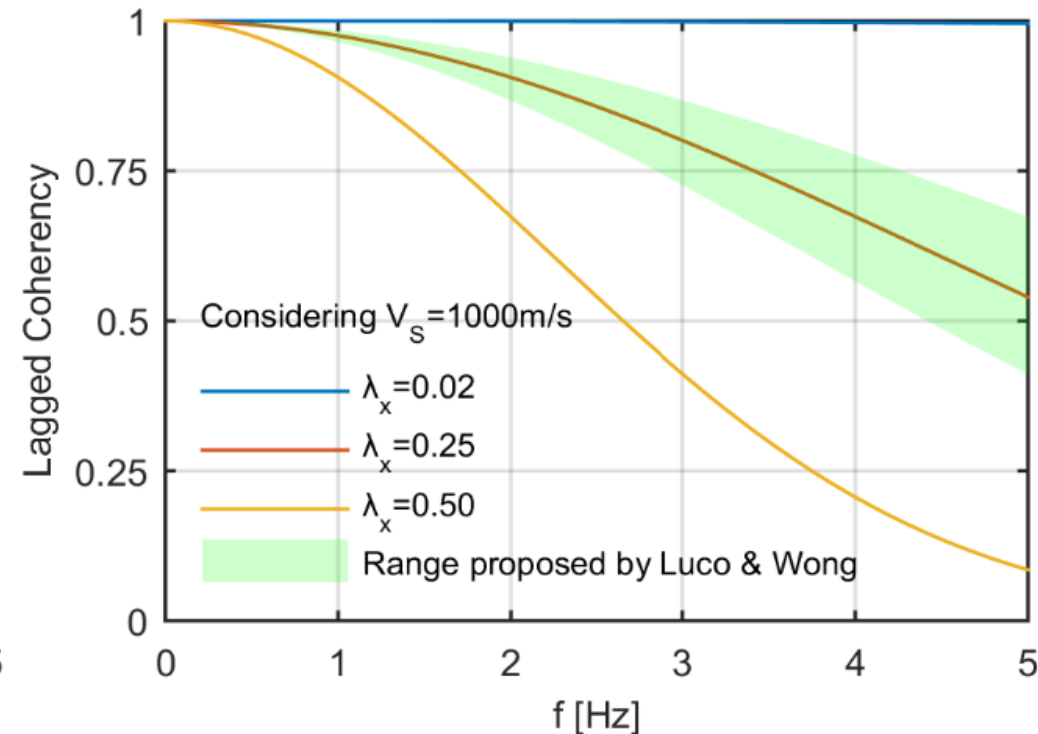
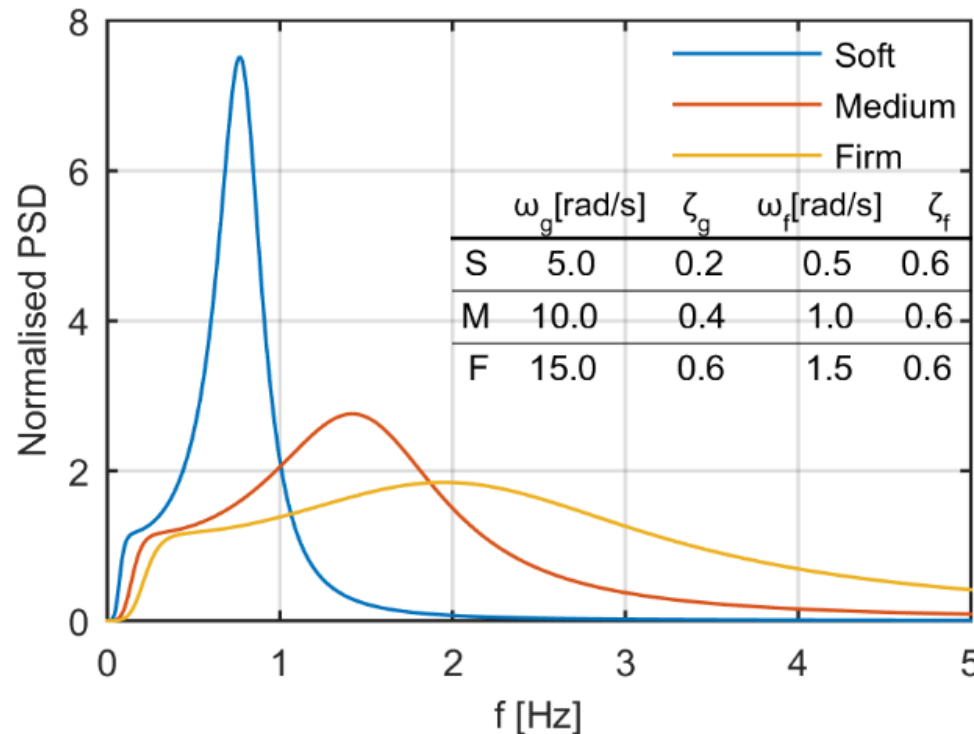
Background of the simplified method for spatial variability in uniform soils

Concept: shift focus from ground motion variability to impact on structural design quantities

- **do not generate samples** of spatially variable ground motions because these are too sensitive to a number of parameters and a probabilistic approach would be required
- **identify the 1st and 2nd antisymmetric modes** of the bridge (which are typically more critically affected by asynchronous motion) and **amplify ONLY the design quantities that are associated with these modes.**

Step 1: Target coherency, intensity and frequency content of ground motions

$$S(\omega) = S_o \frac{\left(1 + 4\zeta_g^2 \left(\omega/\omega_g\right)^2\right)}{\left(1 - \left(\omega/\omega_g\right)^2\right)^2 + 4\zeta_g^2 \left(\omega/\omega_g\right)^2} \frac{\left(\omega/\omega_f\right)^4}{\left(1 - \left(\omega/\omega_f\right)^2\right)^2 + 4\zeta_f^2 \left(\omega/\omega_f\right)^2}$$

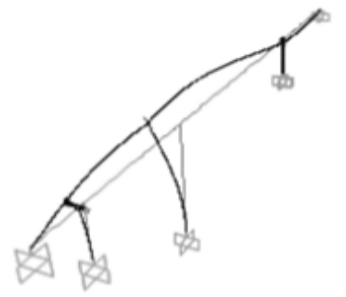


Papadopoulos, S. P., & Sextos, A. G. (2020). Simplified design of bridges for multiple-support earthquake excitation. *Soil Dynamics and Earthquake Engineering*, 131, 106013.

Step 2: Bridge finite element model development - Modal analysis



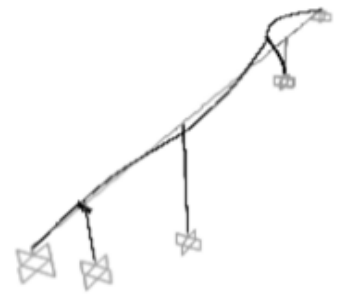
1st Mode
 T=3.78s - f=0.26Hz
 U_y=49.59% R_z=20.58%



3rd Mode
 T=2.10s - f=0.48Hz
 U_y=0.07% R_z=3.37%



6th Mode
 T=1.30s - f=0.77Hz
 U_y=27.36% R_z=57.95%



12th Mode
 T=0.55s - f=1.82Hz
 U_y=1.00% R_z=0.00%



15th Mode
 T=0.40s - f=2.50Hz
 U_y=6.10% R_z=1.70%



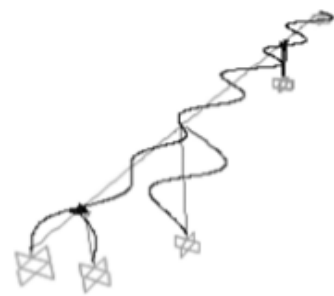
20th Mode
 T=0.30s - f=3.33Hz
 U_y=0.35% R_z=1.12%



29th Mode
 T=0.17s - f=5.88Hz
 U_y=1.00% R_z=0.63%



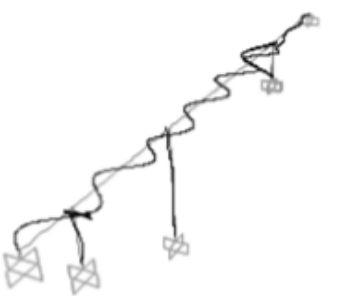
30th Mode
 T=0.15s - f=6.67Hz
 U_y=1.08% R_z=0.08%



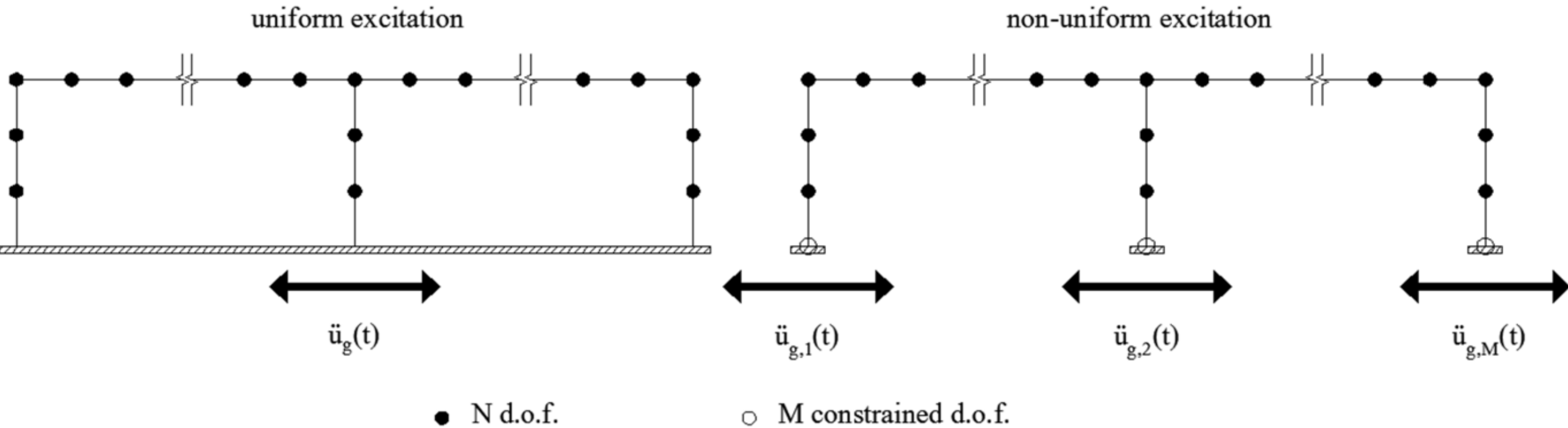
46th Mode
 T=0.09s - f=11.11Hz
 U_y=0.56% R_z=1.93%



47th Mode
 T=0.09s - f=11.63Hz
 U_y=1.94% R_z=2.94%



Step 3: Influence matrix R (M static analyses)



influence matrix [NxM]

$$\mathbf{R} = -\mathbf{K}^{-1}\mathbf{K}_c$$

each column $\{\mathbf{r}_k\}$ of matrix \mathbf{R} represents the static displacements of the structure's unconstrained DOF when its k^{th} support experiences unit displacement while the rest ones

Step 4: Modal participation factors

the modal participation factors for each mode i ($i=1,\dots,N$) associated with the k^{th} support excitation ($k=1,\dots,M$) are calculated by:

$$\Gamma_{i,k} = \frac{\boldsymbol{\varphi}_i^T \mathbf{M} \{\mathbf{r}_k\}}{\boldsymbol{\varphi}_i^T \mathbf{M} \boldsymbol{\varphi}_i}$$

where eigenmodes $\boldsymbol{\varphi}_i$ $\{N \times 1\}$ and mass matrix \mathbf{M} $[N \times N]$ are determined in Step 2, and $\{\mathbf{r}_k\}$ is the k^{th} column of the influence matrix \mathbf{R} computed in Step 3.

Step 5: Non-dimensional spatial variability parameter $\Psi(\omega)$

1. Assign the “reference” ground motion at one of the abutments in the frequency domain $U_o(\omega)$
2. Ground motions at the supports $\mathbf{U}(\omega)$ (forming an $\{M \times 1\}$ array) are described in the frequency domain as a function of the “reference” one:

$$\mathbf{U}(\omega) = \begin{bmatrix} U_1(\omega) \\ \vdots \\ U_M(\omega) \end{bmatrix} = \mathbf{\Psi}(\omega) U_o(\omega) = \begin{bmatrix} \Psi_1(\omega) \\ \vdots \\ \Psi_M(\omega) \end{bmatrix} U_o(\omega)$$

$$\Psi_x(\omega) = \frac{U(\omega)}{U_o(\omega)} = \frac{e^{i\omega R_\theta x + i\frac{\omega\phi_o}{\omega_o}}}{e^{i\frac{\omega\phi_o}{\omega_o}}} = e^{i\omega R_\theta x} = e^{i\omega \sqrt{\frac{2\lambda_x^2}{V_s^2} + \frac{1}{V_{app}^2}} x}$$

$$|\gamma(\xi, \omega)| = \exp\left[-(\lambda_x \omega \xi / V_s)^2\right] \quad \text{where } \lambda_x = \mu(H/r_o)^{0.5}$$

$B_i(\omega)$ is:

- a frequency-dependent complex number for SVFM
- the conventional modal participation factor Γ_i for uniform excitation

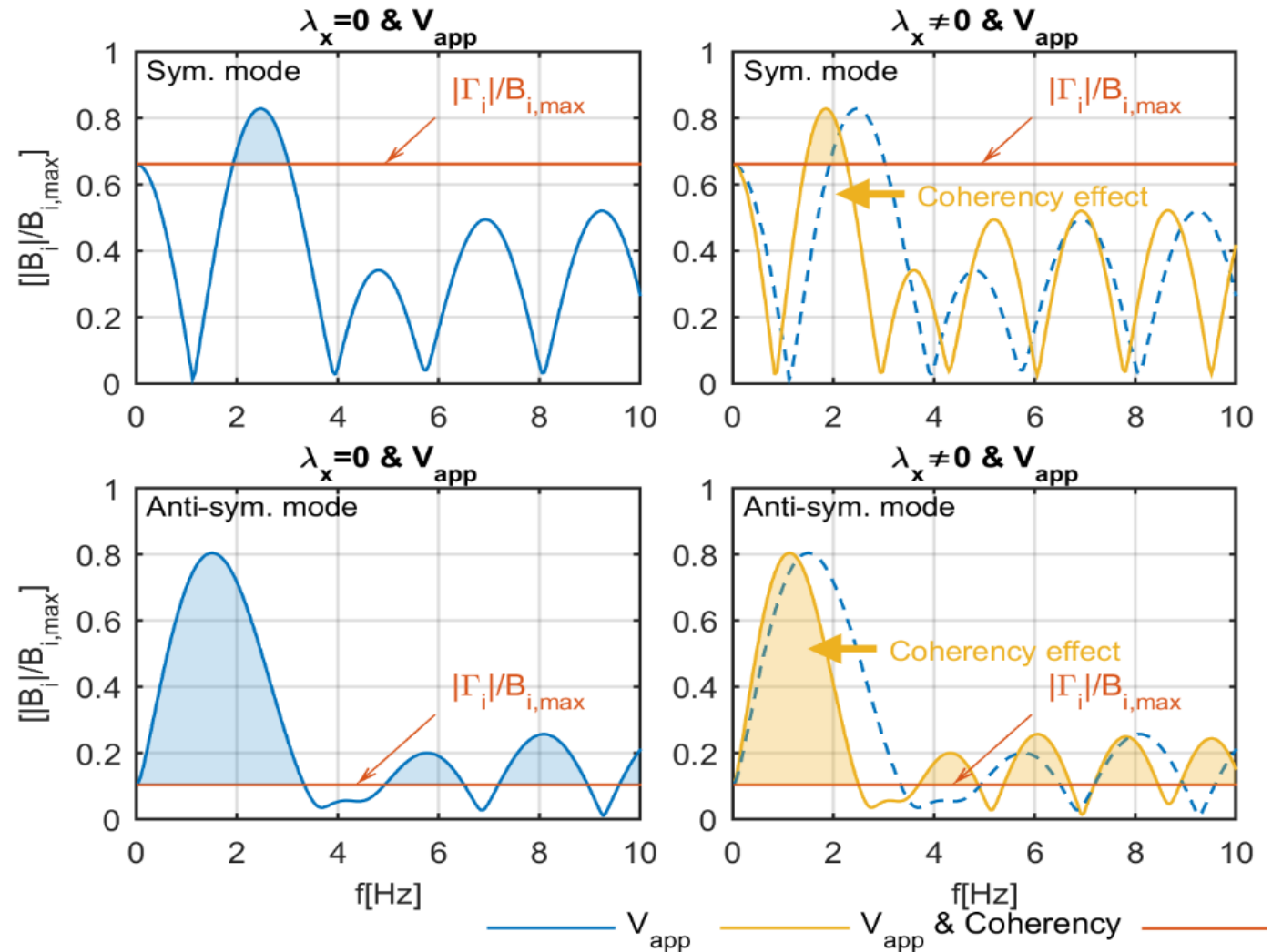
$$B_i(\omega) = (\boldsymbol{\varphi}_i^T \mathbf{M} \mathbf{R}) \boldsymbol{\Psi}(\omega)$$

Since B_i does not depend on the properties of the “actual” support time histories but on the model used to represent SVGM, the ratio:

$$\frac{|B_i(\omega)|}{B_{i,max}} = \frac{|B_i(\omega)|}{\sum_{k=1}^M \left| (\boldsymbol{\varphi}_i^T \mathbf{M} \mathbf{K}^{-1} \mathbf{K}_c)_k \right|}$$

quantifies the potentially amplified contribution of the excited anti-symmetric modes

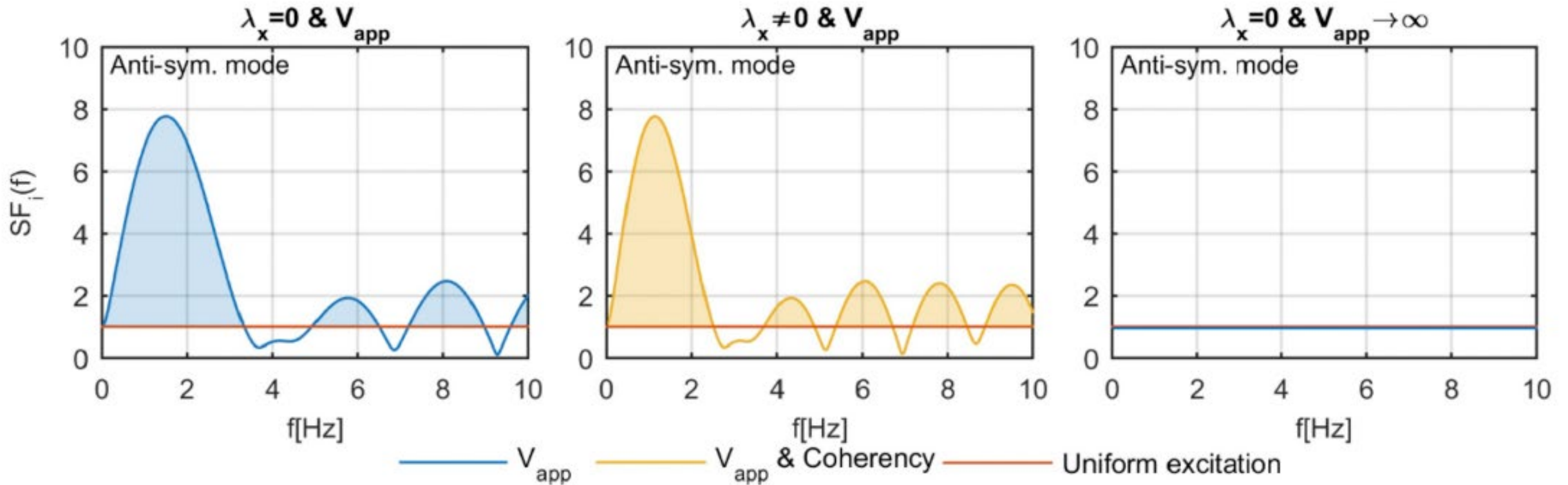
Step 6: Generalized participation factor B_i



Step 7: Frequency-dependent scale factor $SF_i(\omega)$

Scaling factor to account for the amplification of anti-symmetric modes across all frequencies of excitation

$$SF_i(\omega) = \frac{|B_i(\omega)|/B_{i,max}}{|\Gamma_i|/B_{i,max}}$$



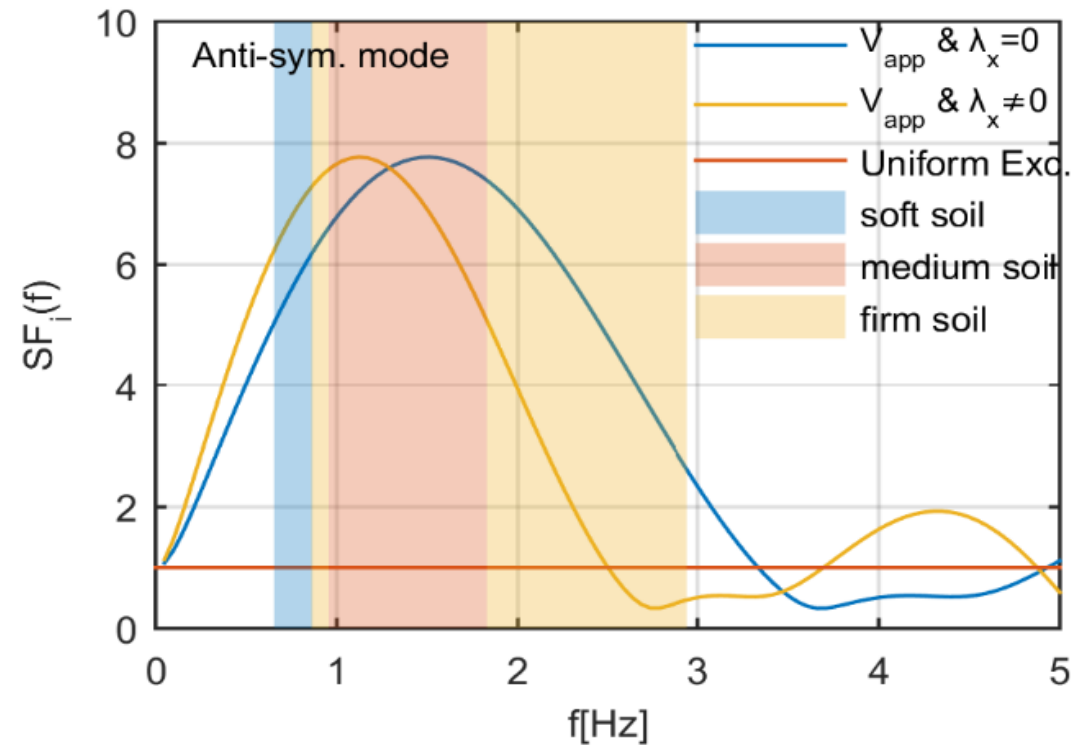
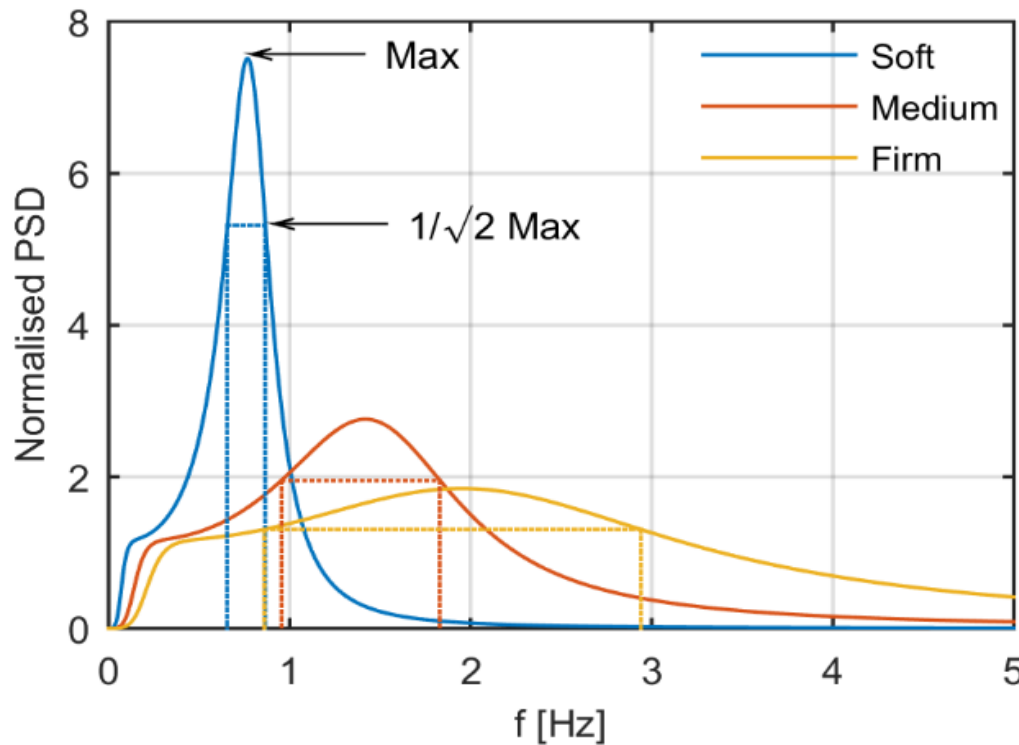
Wave passage only

Wave passage + incoherency

Uniform excitation (SF=1)

Step 8: Frequency range to calculate SF

How to define the excitation range of frequencies across which we can take a mean SF?



Bandwidth for estimating the mean SF in the case of the Clough & Penzien - for (a) firm soil: 0.86-2.94Hz, (b) medium stiffness soil: 0.955-1.83Hz, and (c) soft soil: 0.655-0.865Hz soil (Der Kiureghian & Neuenhofer [11] parameters used for the spectra).

EC8 application of the simplified method for spatial variability in uniform soils

Simplified Implementation in Eurocode 8

1. Run Modal Analysis
2. Run two static analyses using modal load profiles for the first and second antisymmetric mode

$$F_i = \left(\overline{SF}_i - 1 \right) \Gamma_i S_e(T_i) M \phi_i$$

where

F_i is the vector of static forces for the i -th quasi-antisymmetric mode;

T_i is the i -th modal period from modal analysis;

M is the mass matrix;

ϕ_i is the i -th modal shape from modal analysis;

$\Gamma_i = \sum_{k=1}^{N_s} \Gamma_{ik}$ is the i -th modal participation factor due to spatially variable excitation in N_s static modes, obtained as the sum of the participation factors of the i -th mode due to each individual static mode, given in (4);

\overline{SF}_i is a mode amplification factor assumed equal to 4,0 and 2,0, for the first and second quasi-antisymmetric mode, respectively.

□ Set $SF_1=4$ and $SF_2=2$ for the first and second antisymmetric mode

(5.23)

NOTE The product $\Gamma_i S_e(T_i) M \phi_i$ represents the modal forces due to the i -th mode. They do not coincide with those coming from a uniform excitation due to the different value of the participation factor Γ_i .

Simplified Implementation in Eurocode 8

3. Run conventional (uniform) response spectrum or time history analysis
4. Superimpose the design quantities of the above three loading conditions

$$M = \sqrt{M_{conv.}^2 + \sum_i M_{F_i}^2}$$

Thank you for your kind attention

Anastasios Sextos



Further information: www.asextos.net

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