WEBINAR 4: Silos, tanks, pipelines, towers masts and chimneys – Silos and pressure functions of tanks

Christoph Butenweg - SDA engineering GmbH

30th June 2023
RULES FOR SILOS - SCOPE

Section 5 of EN 1998-4 gives rules for the structural analysis and design of steel, reinforced concrete and prestressed precast reinforced concrete silos subjected to seismic actions.

A distinction is made between:

- On-ground
- Elevated silos, supported on a skirt extending to the ground or by substructures

The principles of the seismic analysis procedures may also be applicable for tanks made of other materials (e.g. glass fibre-reinforced plastic/polymer (GFRP), high density polyethylene (HDPE) or polyethylene (PE)).
MODELLING – SILO AND CONTENT

- The silo model should reproduce the strength, the damping, the geometrical properties and the stiffness and mass distribution of the silo, containment structures, external ancillary elements and connecting pipes.

- The silo content should be considered by additional structural masses assuming that the particulate content moves together with the silo shell. The distribution of the masses should reproduce the dynamic effects of the silo content.

- The mass of the content should be determined with EN 1991-4, Annex C, using the upper characteristic value of the bulk unit weight of the particulate solid. Alternatively, the bulk unit weight may be determined by material tests.
MODELLING – FILLING LEVELS

- The dynamic effects should be calculated for the maximum operating filling level.

- In batteries of silos, full, intermediate filled and empty silos should be considered.

- In silos internally subdivided in several cells, the most unfavourable distribution of full, intermediate filled and empty cells should be considered.
STRUCTURAL ANALYSIS

Standard: Force-based approach. Alternatively, response history analysis may be carried out.

- On-ground silos should be modeled using simple beam models with distributed masses.

- Elevated silos should be modelled with their substructures.

- In case of squat or retaining silos shell models for the silo wall and rigidly connected volume elements for the content may be used.
BEHAVIOUR FACTOR

Silo: DC1
- Bolted and welded steel silos
  \( q_R = 1.0, \ q_D = 1.0, \ q_S = 1.2 \)
- Reinforced concrete or prestressed precast reinforced concrete silos:
  \( q_R = 1.0, \ q_D = 1.0, \ q_S = 1.5 \)

Substructure: DC1, DC2 and DC3
- To be applied as given in the relevant parts of EN 1998-1-2

Behaviour factor for the vertical component
- \( \min(q; 1.5) \), where \( q \) is behaviour factor for the horizontal component
REACTION FORCES AND MOMENTS FOR ON-GROUND SILOS

\[ F_{\downarrow b} = S_{\downarrow r} (T_{\downarrow 1h} \, m_{\downarrow S} \, \lambda) \]

\[ F_{\downarrow bv} = S_{\downarrow rv} (T_{\downarrow 1v} \, m_{\downarrow S}) \]

\[ M_{\downarrow b} = S_{\downarrow r} (T_{\downarrow 1h} \, m_{\downarrow S} \, \lambda \, H_{\downarrow f} / 2) \]

\[ \lambda \]
Reduction factor between 0.8 to 1.0

\[ H_{\downarrow f} \]
Maximum filling height

\[ m_{\downarrow S} \]
Total mass of silo and content
NORMAL PRESSURE DISTRIBUTIONS DUE TO HORIZONTAL SEISMIC ACTIONS

Circular silo

\[ \Delta_{ph,s}(\theta) \]

Rectangular silo

\[ \Delta_{ph,s} \]
PRESSURE ON WALLS OF CIRCULAR SILOS – HORIZONTAL ACTIONS

\[ \Delta \phi h_s(\theta) = \Delta \phi h s_0 \cdot \cos \theta \]

Silo wall
Reference pressure
Conical hopper wall

\[ \alpha(z) \cdot \gamma \cdot \min\left( \frac{h b}{d c}, \frac{d c}{2}, 3x \right) \]

Ratio of the response acceleration to the acceleration of gravity
Bulk unit weight
Geometrical parameters

\[ \alpha(z) \cdot \gamma \cdot \min\left( \frac{h b}{d c}, \frac{d c}{2}, 3x \right) \cdot \cos \beta \]

Angle of inclination of the hopper wall

\[ d_{ph}(\theta) \]

Christoph Butenweg
30th June 2023
PRESSURE ON WALLS OF RECTANGULAR SILOS – HORIZONTAL ACTIONS

- On the ‘leeward’ wall normal to the horizontal component of the seismic action:
  \[ \Delta \downarrow \text{ph,s} = \Delta \downarrow \text{ph,so} \]

- On the ‘windward’ wall normal to the horizontal component of the seismic action:
  \[ \Delta \downarrow \text{ph,s} = - \Delta \downarrow \text{ph,so} \]

- On the walls parallel to the horizontal component of the seismic action:
  \[ \Delta \downarrow \text{ph,s} = 0 \]
ACCELERATION PROFILE

Static + Seismic

Acceleration Profile $\alpha(z)$
- constant
- linear
- multimodal

Pressures
Filling pressure + Constant pressure = Total pressure

Filling pressure + Variable acceleration pressure = Total pressure

friction
PRESSURE ON WALLS OF CIRCULAR SILOS - VERTICAL ACTIONS

**Pressure components due to filling**

\[ p_{\text{ls, v}} = p_{\text{static}} \cdot (1 + S_{\text{rv}} \frac{(T_{\text{lv}})}{g}) \]

**Scaling factor**

\[ \beta_{\text{lsc}} \]

\[ p_{\text{ls, v}} \]
DESIGN OF RING STIFFENERS FOR VERTICAL SEISMIC ACTIONS

\[ p_{ls, v} = p_{ls, static} \cdot (1 + S \cdot T / g) \]
VERIFICATION SD LIMIT STATE

- Global stability: Verification of overturning, sliding and uplift.


- Silo shell and hopper: Steel shells and hoppers should be verified according to EN 1993-4-1. Reinforced concrete/prestressed precast reinforced shells should be verified according to EN 1992-3 and EN 1992-1-1.

- Substructures of elevated silos should be verified according to EN 1998-1-2.

- In case of silos with base isolation, the deformation compatibility of the structure and the ancillary elements connecting the silo to the ground or to adjacent structures should be verified.
VERIFICATION SD LIMIT STATE

- Anchorage systems should be designed applying capacity design with sufficient overstrength. A minimum overstrength factor of 1,25 should be used. Anchor systems should provide an adequate resistance.

- Pre-installed anchors should be designed according to EN 1993-1-8.

- Structural connections between the silo structure and ancillary elements or connecting pipes should be designed to remain elastic, considering all relevant overstrength effects.

- Inlets, outlets and further pipes should be verified to accommodate stresses and distortions due to relative displacements, without their functions being impaired.
VERIFICATION DL and OP LIMIT STATE

- The DL limit state may be considered as verified when the silo shell, the hopper, the substructure, the anchorage, connecting pipes and connections of ancillary elements resist the seismic actions in the elastic range.

- At OP limit state it should be verified that strains (or generalised deformations such as drifts) do not exceed values that are acceptable to maintain the function of the silo and associated equipment.

- For OP limit state criteria applicable to the silo and associated equipment should be derived from the analysis of the components the operability of which is required, as well as from the analysis of their supporting systems.
Annex A

Tables for the seismic design of tanks (normative)
Buckling due to hydrodynamic pressures

Elephant foot buckling

Compression failure

Diamond shaped buckling

Shih, 1981

NISEE e-Library

NISEE e-Library
Normalized pressure functions for horizontal seismic excitation

Convective

Impulsive rigid

Impulsive flexible

Top view
Normalized pressure functions for vertical seismic excitation

Impulsive rigid

Impulsive flexible

Top view
EN 1998-4, 1\textsuperscript{st} generation: Response history analysis

Convective pressure distribution due to horizontal excitation

\[ p_c(\xi, \zeta, \theta, t) = \rho \sum_{n=1}^{\infty} \psi_n \cos h (\lambda_n \gamma) J_1(\lambda_n \xi) \cos \theta A_{cn}(t) \]

\[ \psi_n = \frac{2R}{(\lambda_n^2 - 1) J_1(\lambda_n) \cos h (\lambda_n \gamma)} \]

Impulsive rigid pressure distribution due to horizontal excitation

\[ p_i(\xi, \zeta, \theta, t) = C_i(\xi, \zeta) \rho H \cos \theta A_g(t) \]

\[ C_i(\xi, \zeta) = 2 \sum_{n=0}^{\infty} \frac{(-1)^n}{I_1(v_n \gamma) v_n^2} \cos (v_n \xi) I_1\left(\frac{v_n}{\gamma} \xi\right) \]

\[ v_n = \frac{2n + 1}{2} \pi; \quad \gamma = H / R \]

\( I_1: \text{First order Bessel function} \)
**EN 1998-4, 1st generation: Response history analysis**

Impulsive flexible pressure distribution due to horizontal excitation

\[
p_f(\zeta, \theta, t) = \rho H \psi \cos \theta \sum_{n=0}^{\infty} d_n \cos (v_n \zeta) A_{fr}(t)
\]

Iterative solution with powerful numerical tools required!

\[
\psi = \frac{\int_0^1 f(\zeta) \left[ \frac{\rho s}{\rho H} s(\zeta) + \sum_{n=0}^{\infty} b_n' \cos (v_n \zeta) \right] d\zeta}{\int_0^1 f(\zeta) \left[ \frac{\rho s}{\rho H} f(\zeta) + \sum_{n=0}^{\infty} d_n \cos (v_n \zeta) \right] d\zeta}
\]

\[
b_n' = 2 \frac{(-1)^n}{\nu_n^2} \frac{l_1(v_n / \gamma)}{l_1'(v_n / \gamma)}
\]

\[
d_n = 2 \frac{\int_0^1 f(\zeta) \cos (v_n \zeta) d\zeta}{\nu_n} \frac{l_1(v_n / \gamma)}{l_1'(v_n / \gamma)}
\]

\(I_1: \) First order Bessel function
EN 1998-4, 2nd generation: **Force-based approach** instead of RHA

$H$ is the filling height;

$R$ is the radius;

$\zeta = \frac{z}{H}$ is the dimensionless height;

$\gamma = \frac{H}{R}$ is the ratio of filling height to tank radius;
Normalized pressure functions: Convective, impulsive rigid and flexible

\[ p_{j}(\zeta_{\downarrow}, T_{\downarrow 1i}) = C_{j} (\zeta, \gamma) \Gamma_{j} \rho \cos(\theta) S_{a} (T_{\downarrow 1i}) \downarrow \]

- \( C_{j} (\zeta, \gamma) \): Normalized pressure functions: Table A.1 – A.4
- \( \Gamma_{j} \): Participation factors: Table A.6, A.7
- \( S_{a} (T_{\downarrow 1i}) \): Spectral acceleration for first fundamental period

Convective, impulsive rigid and flexible support reactions

\[ F_{j}, \Gamma_{j} \quad m \uparrow S_{a} (T_{\downarrow 1i}) \]

- \( F_{j} \): Base shear and overturning moments coefficients \( C_{F,j}, C_{MW,j} \): Table A.6, A.7
Example: Normalized impulsive flexible pressure functions

\[ C_{lf,h}(\zeta, \gamma) \]

\[ \zeta = \frac{z}{H} \]

<table>
<thead>
<tr>
<th>( \gamma = 0.2 )</th>
<th>( \gamma = 8.0 )</th>
<th>( \gamma = 9.0 )</th>
<th>( \gamma = 10 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>0.95</td>
<td>0.0232</td>
<td>...</td>
<td>0.5406</td>
</tr>
<tr>
<td>0.90</td>
<td>0.0373</td>
<td>...</td>
<td>0.7066</td>
</tr>
<tr>
<td>0.85</td>
<td>0.0479</td>
<td>...</td>
<td>0.7583</td>
</tr>
<tr>
<td>0.80</td>
<td>0.0561</td>
<td>...</td>
<td>0.7558</td>
</tr>
<tr>
<td>0.75</td>
<td>0.0625</td>
<td>...</td>
<td>0.7291</td>
</tr>
<tr>
<td>0.70</td>
<td>0.0673</td>
<td>...</td>
<td>0.6898</td>
</tr>
<tr>
<td>0.65</td>
<td>0.0708</td>
<td>...</td>
<td>0.6452</td>
</tr>
<tr>
<td>0.60</td>
<td>0.0732</td>
<td>...</td>
<td>0.5977</td>
</tr>
<tr>
<td>0.55</td>
<td>0.0747</td>
<td>...</td>
<td>0.5489</td>
</tr>
<tr>
<td>0.50</td>
<td>0.0752</td>
<td>...</td>
<td>0.4995</td>
</tr>
<tr>
<td>0.45</td>
<td>0.0750</td>
<td>...</td>
<td>0.4498</td>
</tr>
<tr>
<td>0.40</td>
<td>0.0742</td>
<td>...</td>
<td>0.4001</td>
</tr>
<tr>
<td>0.35</td>
<td>0.0729</td>
<td>...</td>
<td>0.3502</td>
</tr>
<tr>
<td>0.30</td>
<td>0.0712</td>
<td>...</td>
<td>0.3007</td>
</tr>
<tr>
<td>0.25</td>
<td>0.0692</td>
<td>...</td>
<td>0.2514</td>
</tr>
<tr>
<td>0.20</td>
<td>0.0670</td>
<td>...</td>
<td>0.2030</td>
</tr>
<tr>
<td>0.15</td>
<td>0.0649</td>
<td>...</td>
<td>0.1562</td>
</tr>
<tr>
<td>0.10</td>
<td>0.0629</td>
<td>...</td>
<td>0.1131</td>
</tr>
<tr>
<td>0.05</td>
<td>0.0613</td>
<td>...</td>
<td>0.0773</td>
</tr>
<tr>
<td>0</td>
<td>0.0605</td>
<td>...</td>
<td>0.0594</td>
</tr>
</tbody>
</table>

Christoph Butenweg

30th June 2023
FE-model with stress and buckling verification according to EN 1993

\[ T = 1, \theta, \zeta, T_{1i} \) = \mathbf{C} \mathbf{j} (\zeta, \gamma) \mathbf{F} \mathbf{j} \ \rho \ \lambda \ \cos(\theta) \ \mathbf{S} \mathbf{a} (T_{1i}) \]

Impulsive flexible  
Impulsive rigid  
Convective
Thank you for your attention!